Math425 Final Exam Review Topics

- Definitions of $\max, \min, \sup, \inf$ of subsets of $\mathbb{R}$. Statement of Completeness Axiom. Denseness of rational $\mathbb{Q}$, construction of convergent subsequence of $\mathbb{Q}$ to any given real number.
- Definitions of limits, including $\pm\infty$ cases. Show limits by definition. Proofs of: convergence $\Longrightarrow$ boundedness; monotone sequences always have limits, both bounded and unbounded cases.
- Definitions of $\limsup, \liminf$, Cauchy sequences, properties of Cauchy sequences. Equivalence definitions of limits and proofs: $\lim s_n = s \Longleftrightarrow \liminf s_n = \limsup s_n = \lim s_n \Longleftrightarrow (s_n)$ is Cauchy in the case of $\lim s_n = s \in \mathbb{R}$.
- Subsequences: Convergence $\Longrightarrow$ subsequential convergence to the same limit; $\exists$ convergent subsequences to $\limsup s_n, \liminf s_n$.
- Bolzano-Weierstrass Theorem, proof of: Every bounded sequences has convergent subsequence. Properties of subsequential limit set S and proofs of: $S \neq \emptyset$; $\liminf s_n = \inf S, \limsup s_n = \sup S$; S is closed: i.e. $t_n \in S$ and $t_n \to t$ than $t \in S$.
- Statement and application of $\liminf \left|\frac{s_{n+1}}{s_n}\right| \leq \liminf |s_n|^{1/n} \leq \limsup |s_n|^{1/n} \leq \limsup \left|\frac{s_{n+1}}{s_n}\right|$.
- Definition of convergent infinite series, Cauchy criterion. Comparison test, absolute convergence implies convergence, applications and proofs of. Ratio Test, Root Test, Integral Test, Alternating Series Test, applications and proofs of.
- Definition of continuity, ϵ-δ equivalence and proof of. Use definition to prove/disprove continuity. Max. and min theorem of continuous functions on closed intervals; Intermediate Value Theorem; Inverse Function Theorem.
- Definition of uniform continuity, ϵ-δ equivalence and proof of. Use definition to prove/disprove uniform continuity. Proof and application of Theorem 19.6. Proofs of various properties of uniform continuity: Theorems 19.2,4,5.
- Definition of limits of functions along any subsets of real numbers, ϵ-δ equivalences and proofs of.
- Power series, statement and proof of radius of convergence by root test, convergence/divergence at end points of intervals of convergence by integral, alternating tests.
- Definitions of pointwise and uniform convergence. Cauchy criterion of uniform convergence and proof of. Use definition to prove/disprove uniform convergence, and technique of Remark 24.4. Statements and proofs of properties of uniformly convergent continuous functions, integrable functions. Proofs and applications to infinite series of functions: Cauchy criterion, Weierstrass M-test (Weierstrass dominant convergent theorem).
- Continuity, integrability, differentiability of power series, i.e. statements and proofs of Theorems 26.1,26.4,26.5. Statement and proof of Abel' Theorem. Statement of Weierstrass's Theorem.
- Definition of derivative, proof by definition. Statements and proofs of differentiation rules. Statements and proofs of Extrema Theorem (29.1), Rolle's

Theorem, Mean Value Theorem, Intermediate Value Theorem of Derivative, Derivative of Inverse Functions. Applications (corollaries) of Mean Value Theorem.

• Statements and proofs of Taylor's Theorems (31.3,31.5). Applications of Taylor's Theorem to elementary functions such as: $e^x, \sin x, \cos x$.